WHAT ARE WE DOING TOGETHER . . .

Classification of discontinuity of a function at a point

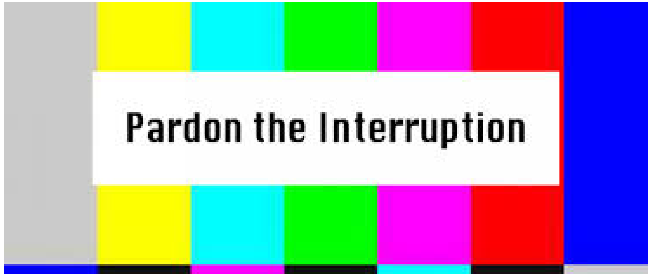
At the end of IV or at the beginning of V class. Hours from 6 to 10.
The presentation is only a guideline to stimulate the speaking.
At first an informal speaking is expected. From the little treasure of these first words the teacher try to lead to a more formal and mathematically talking.
During the lesson, in the slideshows there are gaps instead of the gray words that are only one of the possible choices.

Lesson 1.

https://youtu.be/SIBaF9lrqVY

What do you think about it?

Let’s note down everything you say.



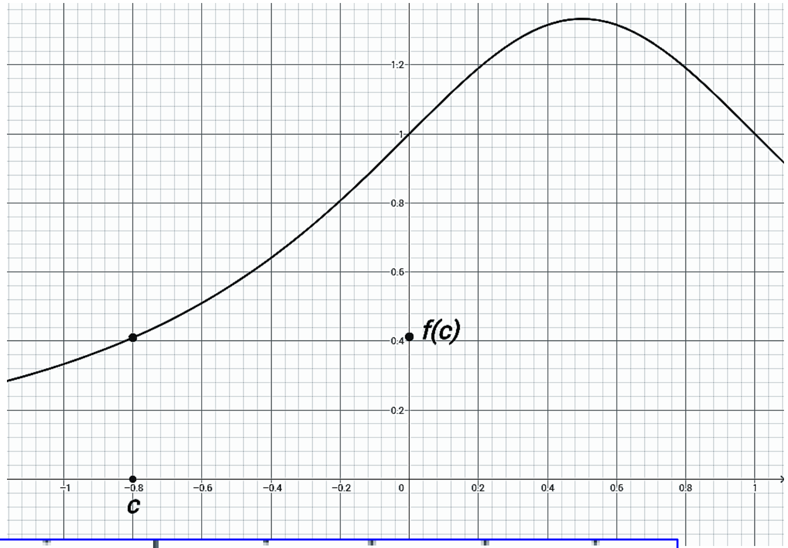
Barbara Serafino

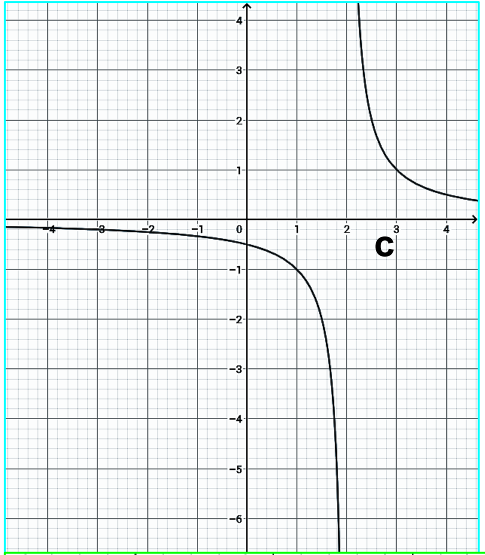
Barbara Serafino

On the contrary . . .

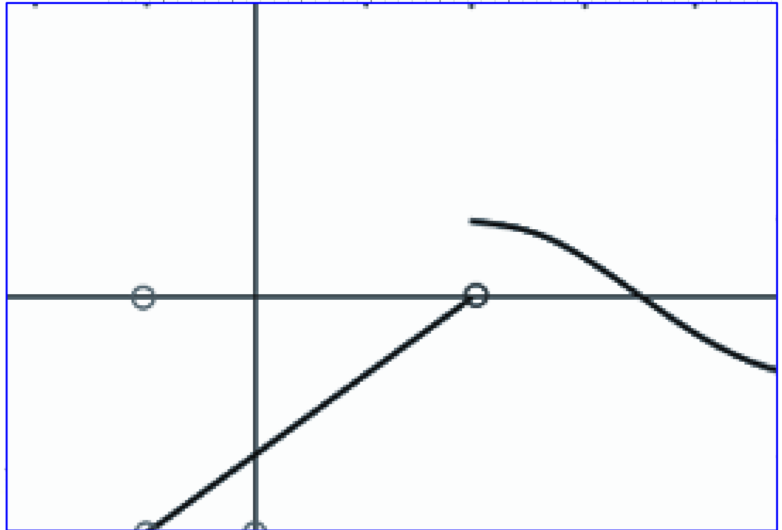
https://youtu.be/PH4cYOupAlY

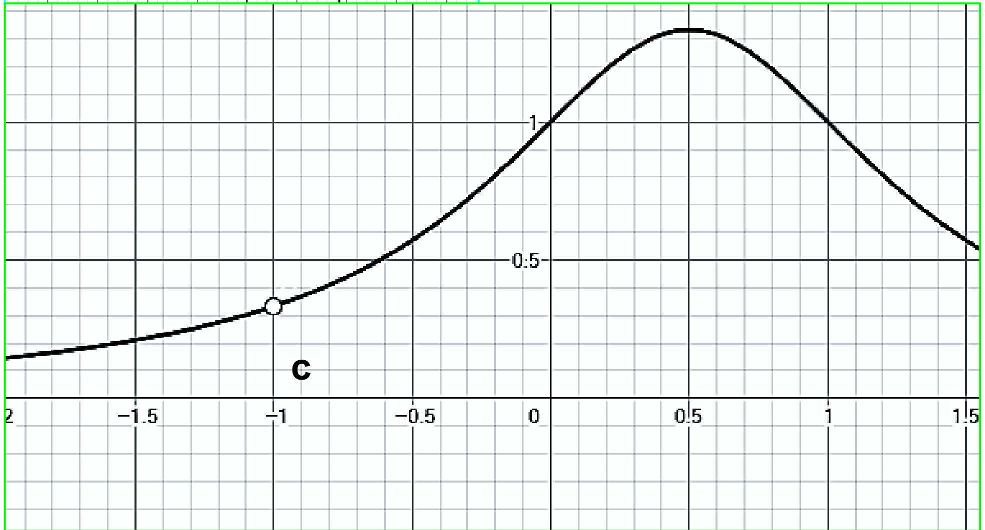
... note down everything you think about these others.



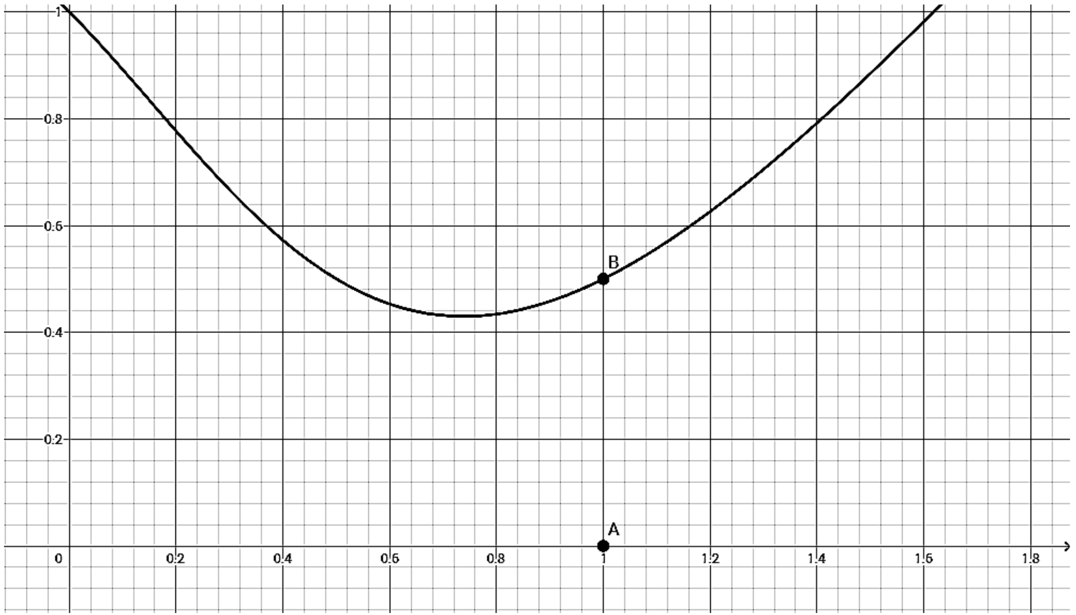


Look at these graphs: how could you draw them with your pencil?





A function is *continuous* at a point if you can draw the *graph* at that point without picking the pencil up!



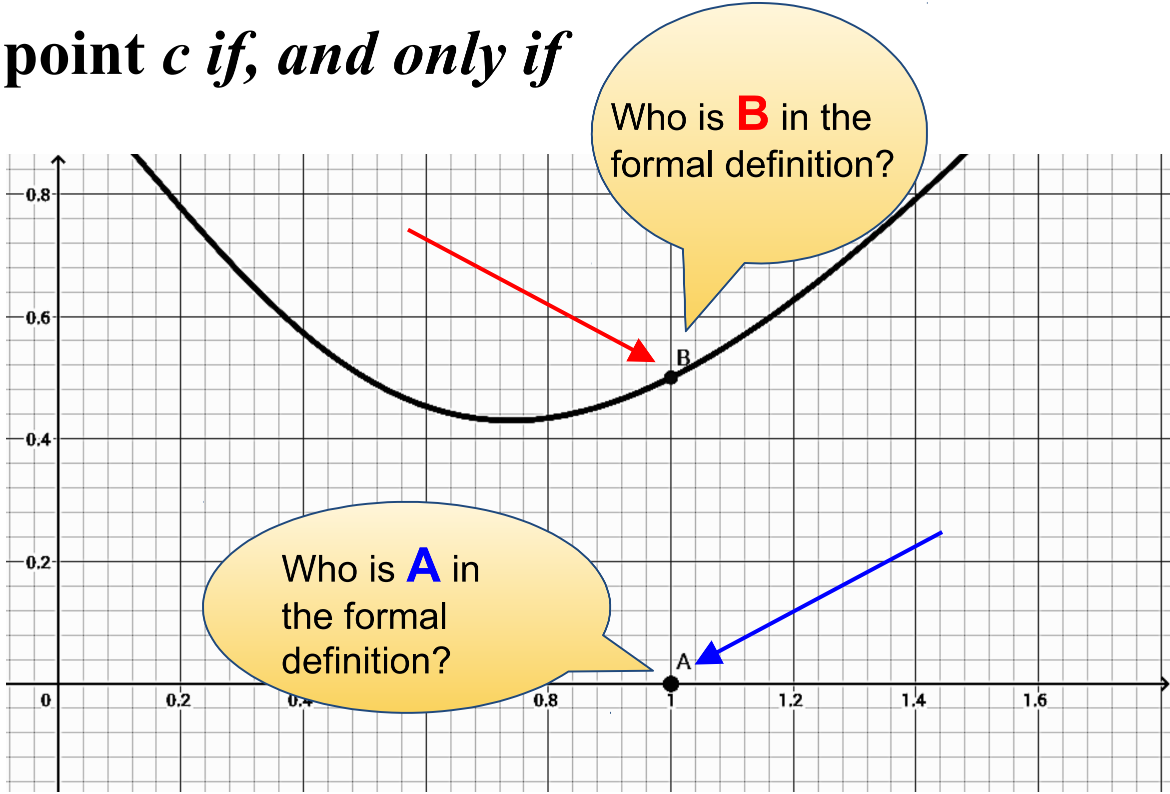
DA VINCI - RIPAMONTI
ISTITUTO ISTRUZIONE SUPERIORE

Barbara Serafino

Formal DEFINITION

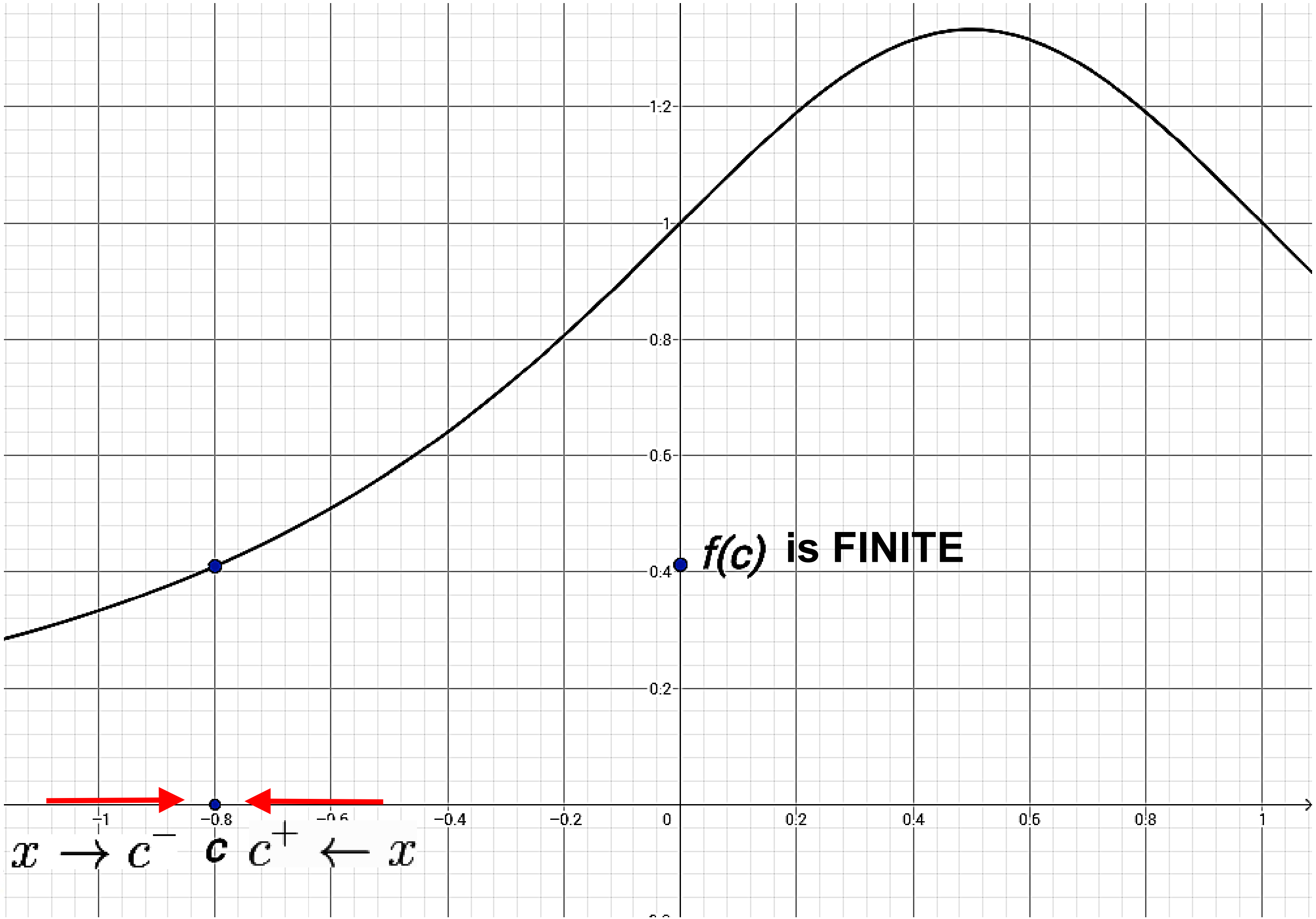
f(x)* is *continuous* at some point *c if, and only if

1. *f(x) exists in c*
2. *the limit of f in c exists*
3. $\lim_{x \to c} f(x) = f(c)$



DA VINCI - RIPAMONTI
ISTITUTO ISTRUZIONE SUPERIORE

Barbara Serafino



The *limit* as $\boldsymbol{x}$ approaches to c from the left is equal to $\boldsymbol{f(c)}$:

$$\lim_{x \to c^{-}} f(x) = f(c)$$

The *limit* as $\boldsymbol{x}$ approaches to c from the right is equal to $\boldsymbol{f(c)}$:

$$\lim_{x \to c^{+}} f(x) = f(c)$$

Both sides:
two-sided limit

Barbara Serafino

Check out it, filling the gaps!

A function *f(x)* is continuous at *c* if,

…………………..

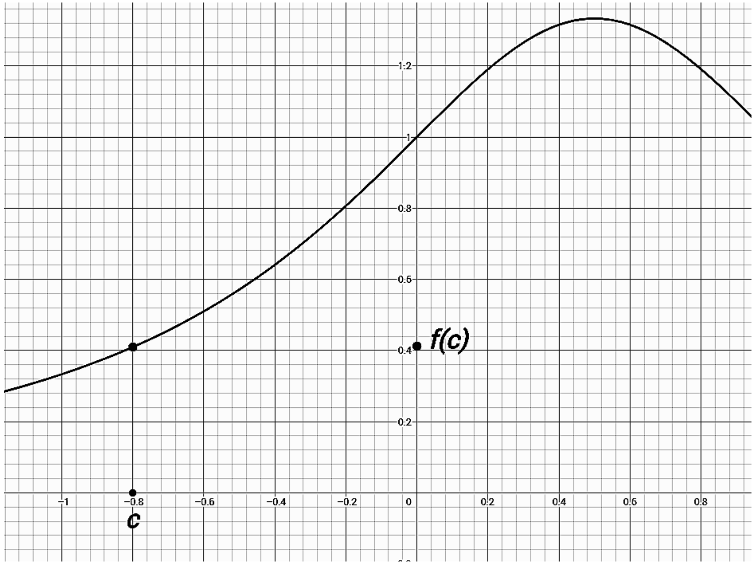
- *f(x)* ……………… in *c*

  (*c* is a point in the Domain of *f(x)*)

- the limit of *f(x)* when *x* ……………………

  to *c* …….

- $\lim_{x \to c} f(x) = \cdots\cdots$

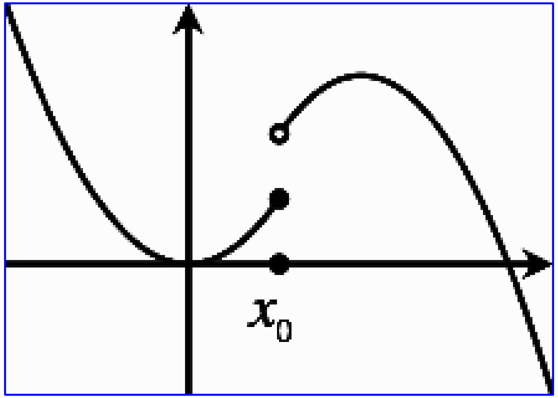


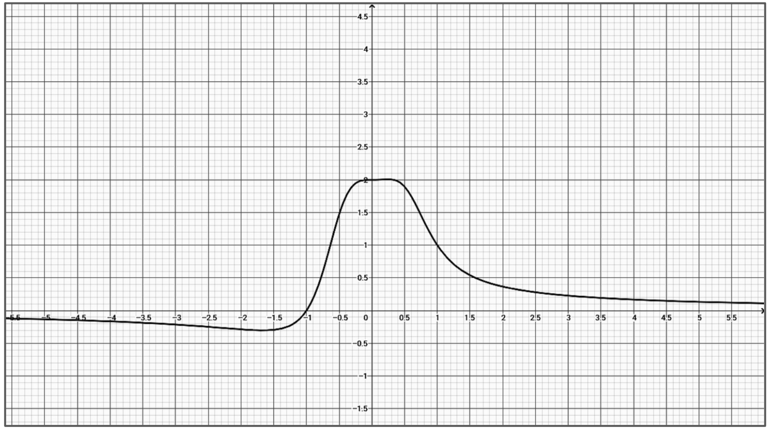
DA VINCI - RIPAMONTI
ISTITUTO ISTRUZIONE SUPERIORE

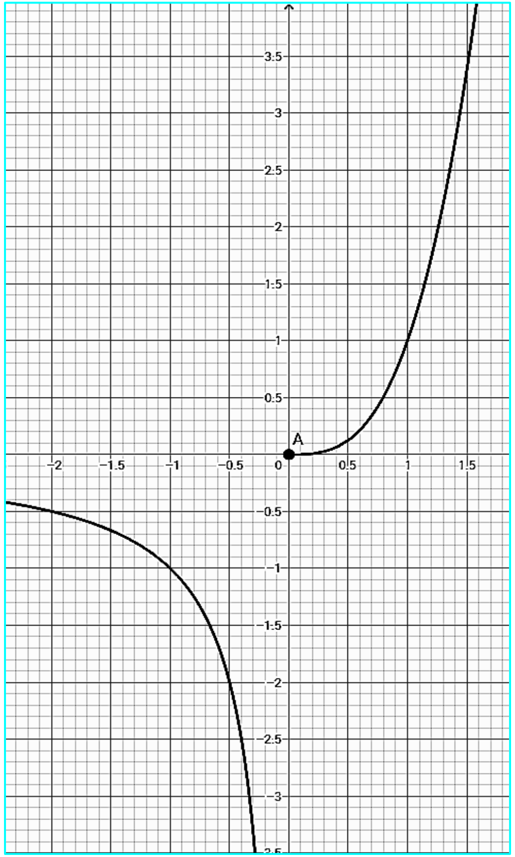
Barbara Serafino

Which are the functions continuous and which are not?

Why?

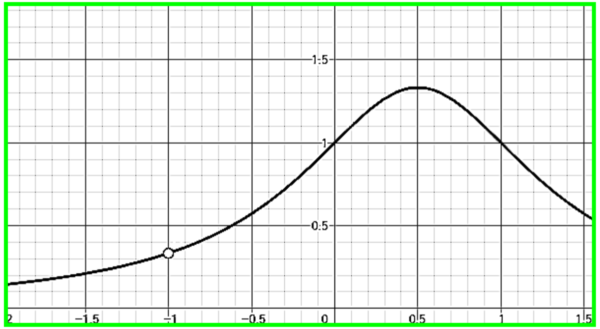


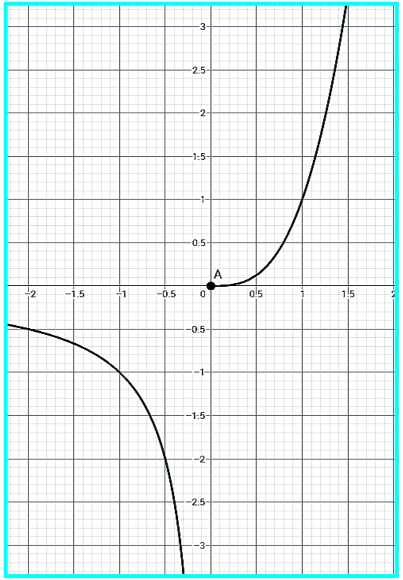


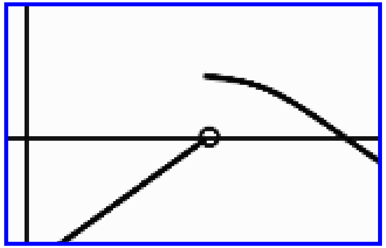


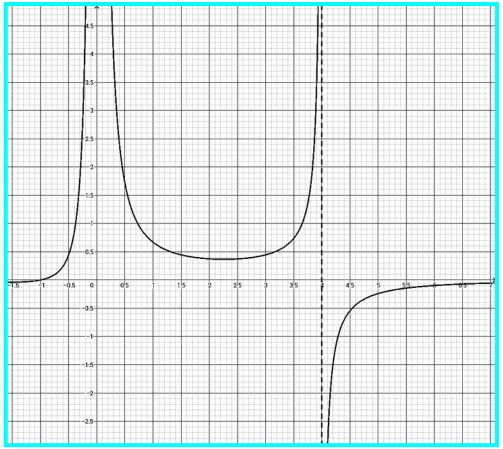
Take turns in talking about the following graphs, using the guide table below.

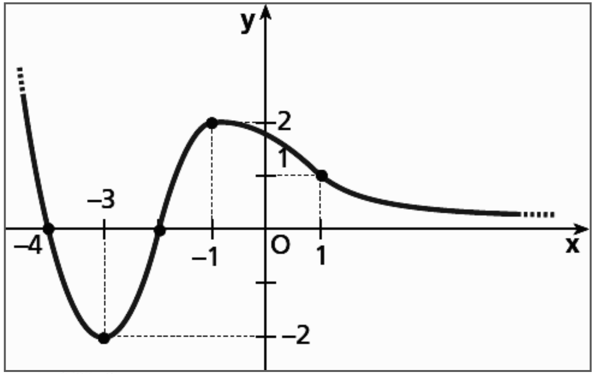
We found that the	graph function	is / is not	continuous discontinuous	at this point because………………………… …………………………………

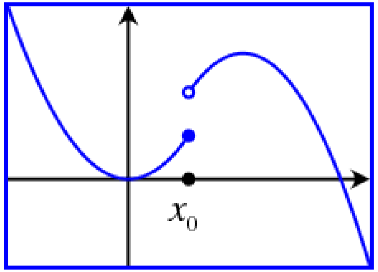


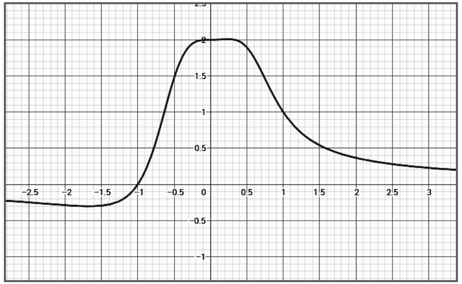












DA VINCI - RIPAMONTI
ISTITUTO ISTRUZIONE SUPERIORE

Barbara Serafino

Classification of discontinuities?

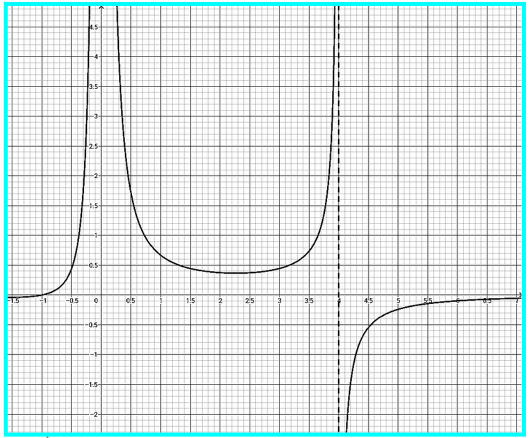
Barbara Serafino

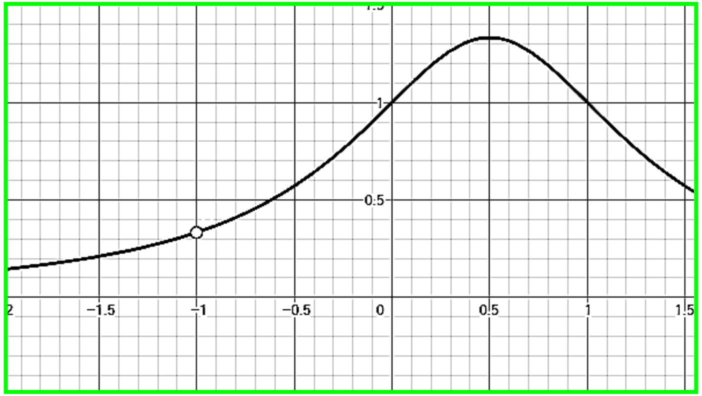
We saw different discontinuities. Look at the graphs below, how should you suggest classifying (putting in different groups) such discontinuities?

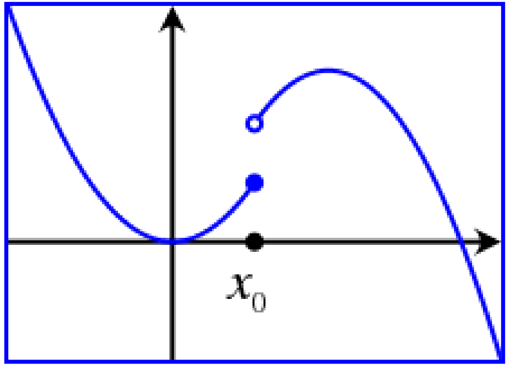
Discuss with your classmate about which of three points of the formal definition is not true in each graph.

Try to explain using the following guide table and note down a sentence for each function.

We can see that the	right / left two-sided one-sided	limit	exists finite exists infinite doesn't exist	because……………….





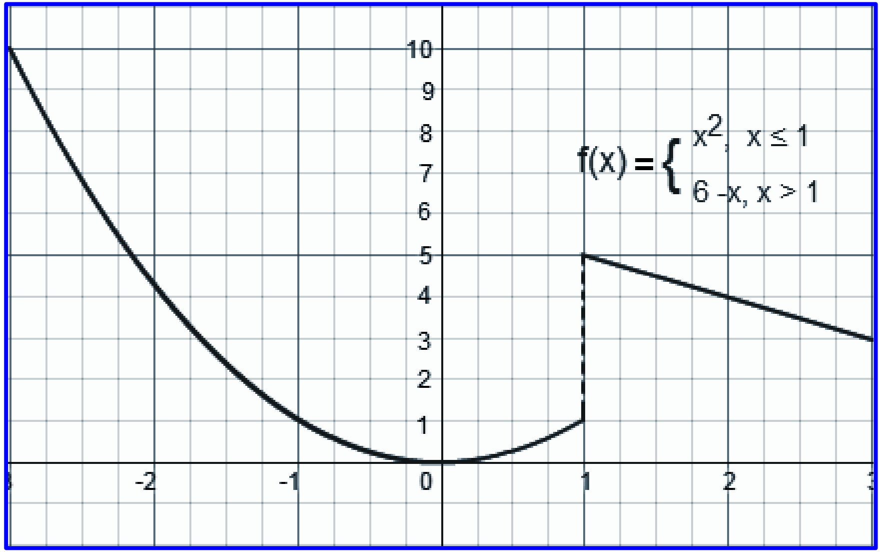


DA VINCI - RIPAMONTI
ISTITUTO ISTRUZIONE SUPERIORE

Barbara Serafino

First kind of discontinuities

Look at the function plotted beside.



Does the right limit exist when x approaches to 1? ________

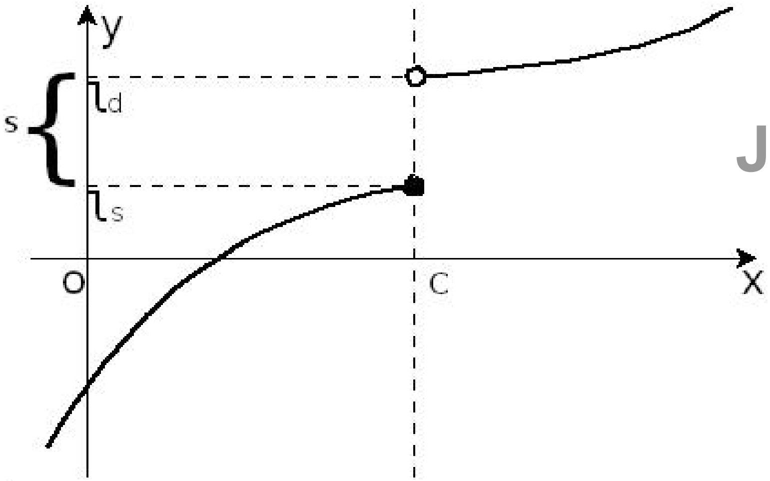
Is it finite? _______

Does the _____ limit exist when x approaches to 1? ________

Is it finite? _______

Both right and left limits exist, they are FINITE and DIFFERENT

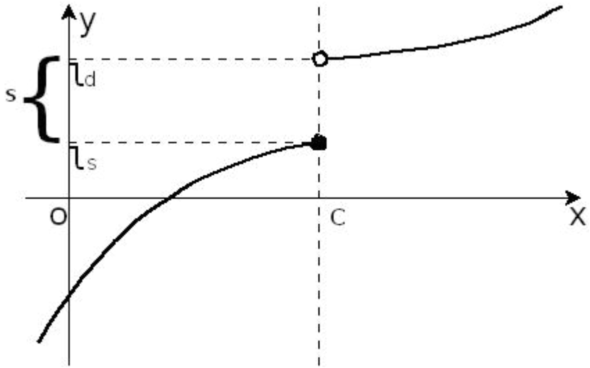
Does the right limit exist when x approaches to c?

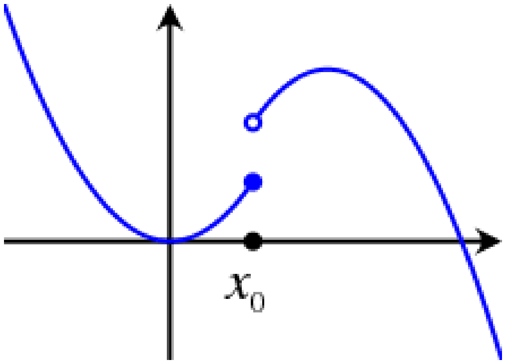


JUMP

The first kind of discontinuity is also called **JUMP DISCONTINUITY**

We can find a Jump Discontinuity in the Piecewise or Hybrid functions

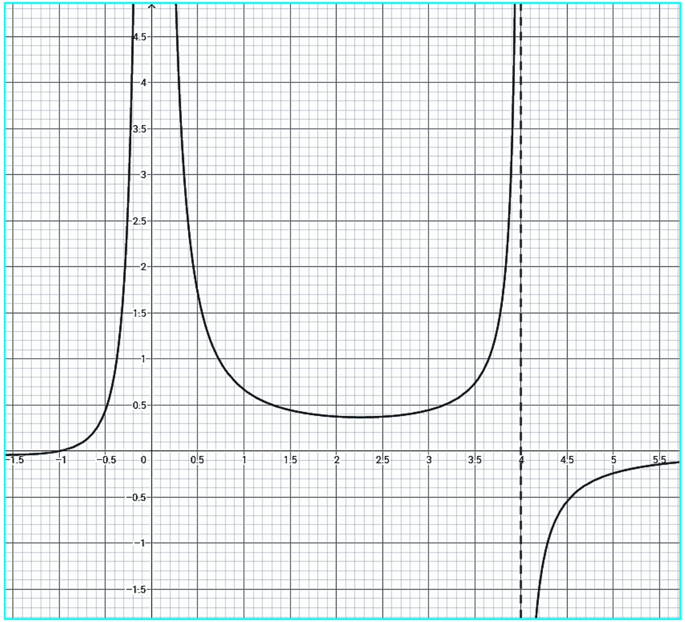


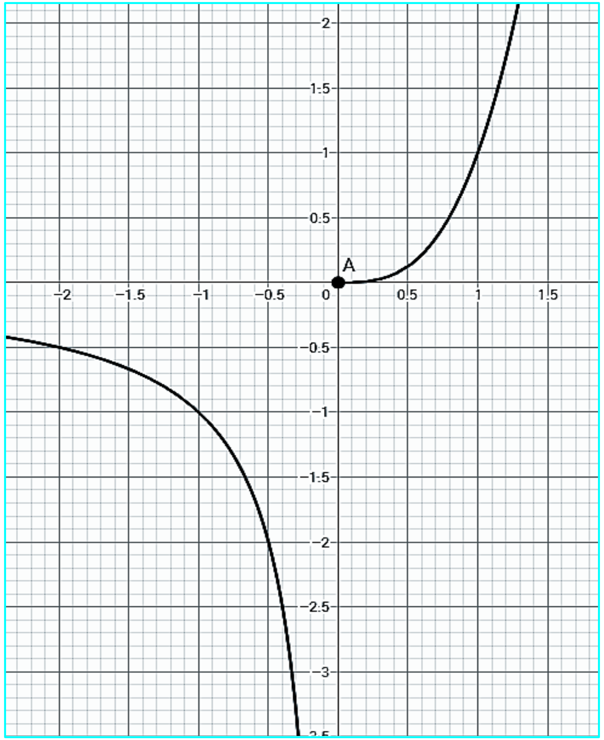


And now some practice for you. I’ll give you some functions.
Find the right and the left limit at the given point and realize if there is a jump discontinuity.

Lesson 4.

The second kind of discontinuities



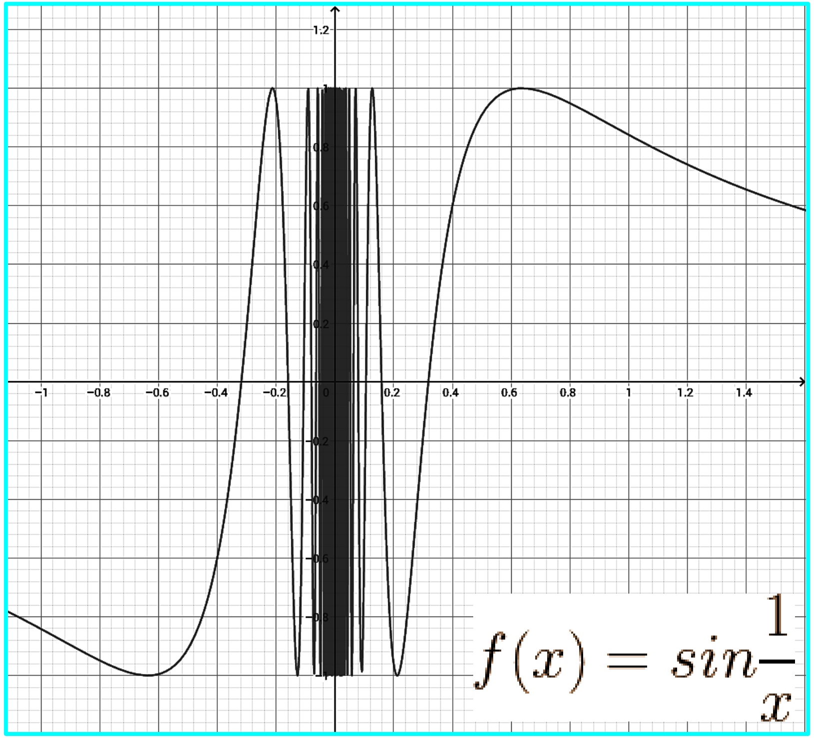


What about the limit at the discontinuity points for these functions?

Try to explain and note down a sentence for each function.

Barbara Serafino

Infinite (asymptotic) or Essential Discontinuities.
One of the one-sided limit is infinite or **doesn’t exist**.



Now, I’ll give some functions from your textbooks.
Identify the points of discontinuities and find if they fall in the first or in the second kind of discontinuity.

Before the next lesson, complete the following table.

ACTIVE	PASSIVE
The teacher asked a difficult question.	
	Discontinuities can be grouped in three different types.
We can draw the graph of a function at a point without any break.	
	The road to school is interrupted by a landslide.
Yesterday the workmen fixed up the interruption on the bridge.	

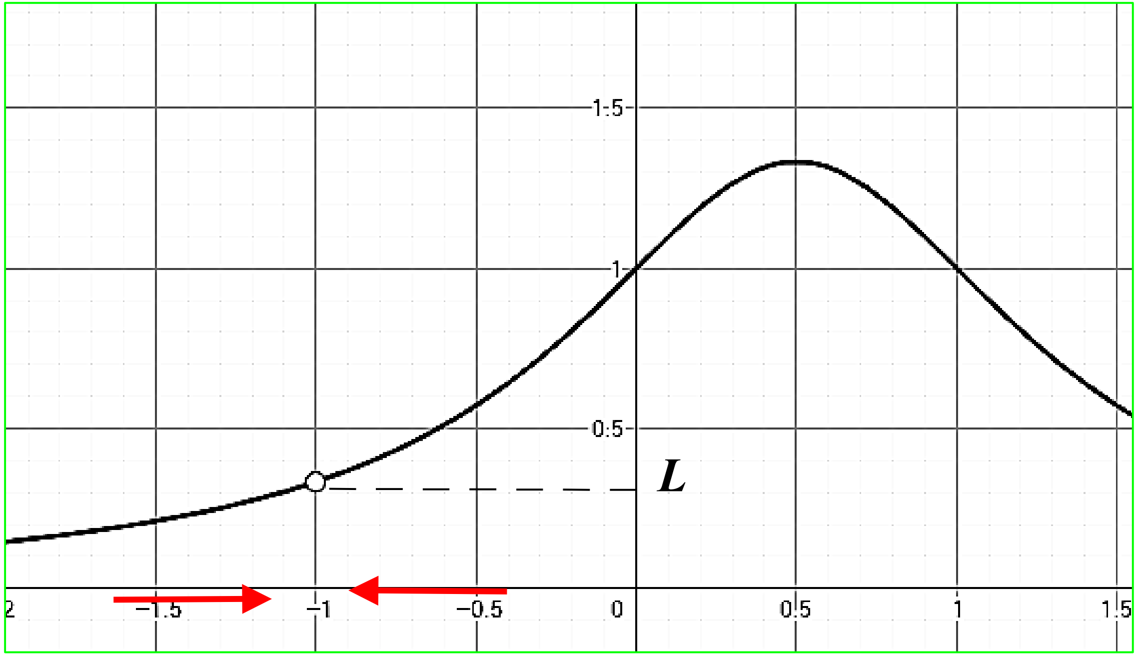
Third kind discontinuities.

The limit exists at the point *c*, but the function doesn't exist in *c*.

In this example *c*=-1, the function is not defined in *c.*

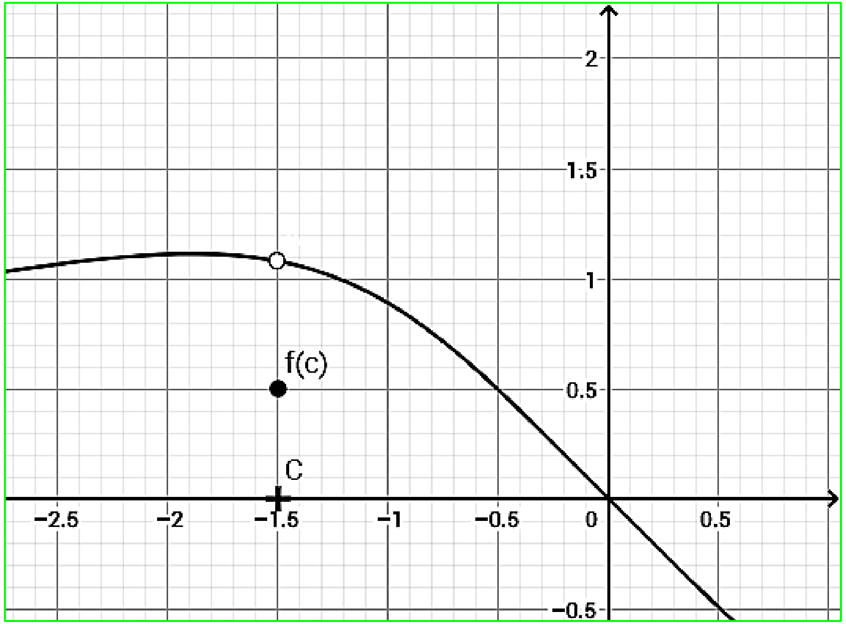
The *limit* as x approaches to *c* from the left is equal to the *limit* as x approaches to c from the right, so

$$\lim_{x \to c} f(x) = L$$



This is also a third kind discontinuity, but what's the difference with the previous one?

Write your thoughts down.



Before continuing, I'll choice some functions from your textbooks.
Your task is to identify and classify the point of discontinuity and provide a graphical interpretation.

Third kind of discontinuities can be **fixed up or removed**, filling the little hole in the graph with **the value of the limit:**

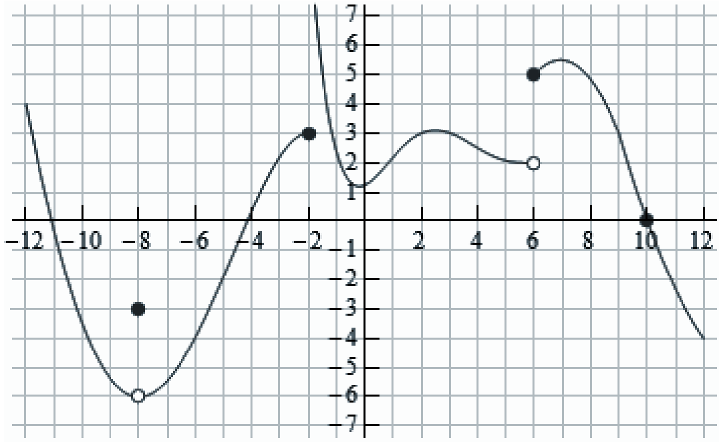
$$\lim_{x \to c} f(x) = L$$

For this reason it is called
REMOVABLE DISCONTINUITY.

WHEN AND HOW A DISCONTINUITY CAN BE FIXED UP.

Exercise.
Let's see how redefining the function $y = \frac{x^2 - 3x + 2}{x^2 - 1}$ in order to *remove* a *removable* discontinuity at the point *c*=−1.

Look at the graph below and match the **black** points to the correct label.



A. Infinite (asymptotic) discontinuity
B. Jump Discontinuity
C. Continuity point
D. Removable discontinuity

Now, look at the graph above and determine the limits of *f(x)* for *x* that approaches to the **black** points.

$$\lim_{x \to \dots} f(x) = \dots$$

$$\lim_{x \to \dots} f(x) = \dots$$

$$\lim_{x \to \dots} f(x) = \dots$$

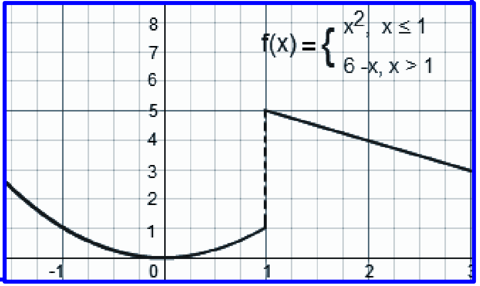
$$\lim_{x \to \dots} f(x) = \dots$$

$$\lim_{x \to \dots} f(x) = \dots$$

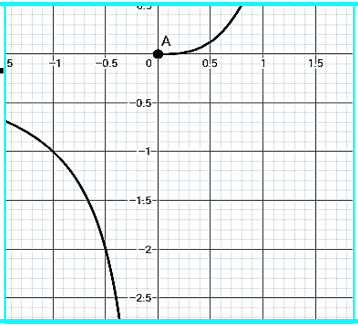
$$\lim_{x \to \dots} f(x) = \dots$$

Types of discontinuities

First kind-**Jump** Discontinuities.
Both left and right limits exist,
but have **finite different** values.



Second kind-**Infinite** (asymptotic) or **Essential** Discontinuities.
One of the two-sided limit **is infinite**
or doesn't exist.



Third kind-**Removable** discontinuities.
Tthe limit exists and is finite, but, at that point,
the function doesn't exist or its value is not equal to the limit.